Julius Wess   Evgeny A. Ivanov (Eds.)

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On Superconformal-Like Transformations and Their Nonlinear Realization

Steven Duplij

Theory Group, Nuclear Physics Laboratory,
Kharkov State University, Kharkov 310077, Ukraine
E-mail: Steven.A.Duplij@univer.kharkov.ua
Internet: http://www-home.univer.kharkov.ua/~duplij

Abstract. We consider nonlinear realizations of invertible and noninvertible $N = 1$ superconformal-like transformations by means of the odd curve motion technique and introduced diagrammatic method.

First papers on supersymmetry (Volkov and Akulov (1972), Volkov and Akulov (1973), Akulov and Volkov (1974)) were written in terms of nonlinear realizations (for nonsupersymmetric background of the method see (Volkov (1969), Volkov (1973), Coleman, Wess, and Zumino (1969))). Further, there were hopes that in the framework of nonlinear realizations one could solve the problems with superpartners and spontaneously supersymmetry breaking in realistic models (Samuel and Wess (1983)). From another side, nonlinearly realized two dimensional superconformal symmetry (Kunitomo (1995)) were used in the theory of superstring embeddings (Belucci, Gribanov, Ivanov, Krivonos, and Pashnev (1998), Berkovits and Vafa (1994)). Here we consider finite superconformal-like (Duplij (1997)) transformations and include noninvertibility (Duplij (1991), Duplij (1997)). We also consider the connection between "linear" and "nonlinear" realizations (Ivanov and Kapustnikov (1978), Ivanov and Kapustnikov (1982)), but from the pure kinematical viewpoint and give a transparent diagram presentation for it in our special case.

Motion of odd curve in $\mathrm{C}^{1|1}$. Let us consider $N = 1$ superanalytic transformations in $\mathrm{C}^{1|1}$

$$\begin{cases} \tilde{z} = f(z) + \theta \cdot \chi(z), \\ \tilde{\theta} = \psi(z) + \theta \cdot g(z), \end{cases} \tag{1}$$

where four component functions $f(z), g(z) : \mathrm{C}^{1|0} \to \mathrm{C}^{1|0}$ and $\psi(z), \chi(z) : \mathrm{C}^{1|0} \to \mathrm{C}^{0|1}$ satisfy supersmooth conditions generalizing C^{∞} (Rogers (1980), De Witt (1992)). According to the interpretation (Wess (1984)) we can study the motion of the curve $\theta = \lambda(z)$ in $\mathrm{C}^{1|1}$. Then we obtain

$$\tilde{z} = f(z) + \lambda(z) \cdot \chi(z), \tag{2}$$

$$\tilde{\lambda}(\tilde{z}) = \psi(z) + \lambda(z) \cdot g(z), \tag{3}$$

where the second equation reflects the Einstein style of transformations.

In four dimensional case the function $\lambda(z)$ is usually called Akulov-Volkov field (Wess (1984)) and in physical applications it plays a role of *Nambu-Goldstone fermion* (Volkov and Akulov (1973), Akulov and Volkov (1974)) (and therefore it is also called a *goldstino*).

The transformation of goldstino $\lambda(z)$ is highly nonlinear as it is seen from (3). Relations of such kind always appear in nonlinear group realizations, and $\lambda(z)$ is responsible for supersymmetry breaking (Akulov and Volkov (1974)).

To find goldstino finite transformations in our case we expand $\tilde{\lambda}(\tilde{z})$ in series and iterate exploiting nilpotency

$$\tilde{\lambda}(f(z)) = \psi(z) + \lambda(z)\cdot g(z) - \tilde{\lambda}'(f(z))\cdot\lambda(z)\cdot\chi(z)\,. \tag{4}$$

In case f^{-1} exists, we derive the finite superanalytic transformation of $\lambda(z)$ as $\tilde{\lambda} = \psi\circ f^{-1} + \lambda\circ f^{-1}\cdot g\circ f^{-1} - \tilde{\lambda}'\cdot\lambda\circ f^{-1}\cdot\chi\circ f^{-1}$,where $f\circ g = f(g(z))$.

It is not possible to find a general solution of the equation (4), and therefore we consider some particular cases.

Global SUSY. The global supersymmetry in $\mathrm{C}^{1|1}$ corresponds to the following choice $f(z) = z$, $g(z) = 1$, $\chi(z) = \varepsilon$, $\psi(z) = \varepsilon$,where ε is a constant odd parameter. Then from (2) and (3) we have

$$\tilde{\lambda}_{Glob}(z) = \varepsilon + \lambda(z) - \tilde{\lambda}'_{Glob}(z)\cdot\lambda(z)\cdot\varepsilon\,. \tag{5}$$

This equation is also difficult to solve manifestly without any additional requirements. But for infinitesimal transformations we obtain

$$\delta_\varepsilon\lambda_{Glob}(z) = \tilde{\lambda}_{Glob}(z) - \lambda(z) = \varepsilon\cdot[1 + \lambda(z)\cdot\lambda'(z)] \tag{6}$$

which satisfy the conventional supersymmetry algebra $[\delta_\varepsilon, \delta_\eta]\,\lambda_{Glob}(z) = 2\varepsilon\eta\cdot\lambda(z)\cdot\lambda'(z)$ in accordance with (Akulov and Volkov (1974)).

In finite global case we put $\tilde{\lambda}^{fin}_{Glob}(z) = \tilde{\lambda}_{Glob}(z) + \Delta(z)$,where $\tilde{\lambda}_{Glob}(z)$ is given by (6). Inserting it into (5) one derives the equation for $\Delta(z)$ as follows $\Delta'(z)\cdot\varepsilon\cdot\lambda(z) = \Delta(z)$ which can be solved by expanding on nilpotents in a given underlying superalgebra.

Let us consider *superconformal-like transformations* parametrized by two functions $g(z)$, $\psi(z)$ (see (Duplij (1996), Duplij (1997))). Then starting from the same function $\lambda(z)$ we can in general find $\tilde{\lambda}_n(z)$ from (3) as two separate solutions (corresponding to the projection n of the "reduction spin" (Duplij (1997))) of the following system of equations

$$\begin{cases} \tilde{\lambda}_n\left(f_n^{(g\psi)}(z)\right) & = \psi(z) + \lambda(z)\cdot g(z) - \tilde{\lambda}'_n\left(f_n^{(g\psi)}(z)\right)\cdot\lambda(z)\cdot\chi_n^{(g\psi)}(z)\,, \\ f_n^{(g\psi)\prime}(z) & = \psi'(z)\,\psi(z) + \frac{1+n}{2}g^2(z)\,, \\ \chi_n^{(g\psi)\prime}(z) & = g'(z)\,\psi(z) + ng(z)\,\psi'(z)\,, \end{cases} \tag{7}$$

where prime denotes derivative by argument, $n = +1$ corresponds to SCf transformations and $n = -1$ - to TPt transformations, and TPt are so called *noninvertible transformations twisting parity of tangent space* (see (Duplij (1991), Duplij (1997))). We will call $\tilde{\lambda}_{SCf}(z) = \tilde{\lambda}_{n=+1}(z)$ a *SCf goldstino*, and $\tilde{\lambda}_{TPt}(z) = \tilde{\lambda}_{n=-1}(z)$ a *TPt goldstino.*

It is necessary to stress that equations (7) do not depend on invertibility properties of superconformal-like transformations (Duplij (1996), Duplij (1991)) and only they can be used to find TPt goldstino evolution ($n = -1$ case). As previously, it is not possible to solve the system (7) manifestly in general case.

Infinitesimal SCf. Let we parametrize infinitesimal SCf transformations by $f(z) = z + r(z)$, $g(z) = 1 + \frac{1}{2}r'(z)$, $\chi(z) = \varepsilon(z)$, $\psi(z) = \varepsilon(z)$, where $r(z), \varepsilon(z)$ are infinitesimal. Then, from (7) we obtain

$$\delta_{r,\varepsilon}\lambda_{SCf}(z) = \varepsilon(z) \cdot [1 + \lambda(z) \cdot \lambda'(z)] + \frac{1}{2}r'(z) \cdot \lambda(z) - r(z) \cdot \lambda'(z) \quad (8)$$

in agreement with (Kunitomo (1995)).

Connection between linear and nonlinear realizations from diagrammatic viewpoint. Relationship between linear and nonlinear realizations (Ivanov and Kapustnikov (1978)) plays an important role in understanding of the spontaneously supersymmetry breaking mechanisms (Ivanov and Kapustnikov (1982)). Here we investigate that in noninvertible finite case and from some another kinematical viewpoint using a clear diagrammatic approach (which is applicable to any general multidimensional case as well). Let us consider the following diagram

$$\begin{array}{ccc} Z_A & \xrightarrow[\text{W-Z}]{\mathcal{G}} & \tilde{Z} \\ \mathcal{A}\Big\uparrow & & \Big\uparrow\mathcal{B} \\ Z & \xrightarrow[\text{A-V}]{\mathcal{H}} & Z_H \end{array}$$

where $\mathcal{A} : Z \to Z_A$, $\mathcal{G} : Z_A \to \tilde{Z}$, $\mathcal{B} : Z_H \to \tilde{Z}$, $\mathcal{H} : Z \to Z_H$ (and $Z = (z, \theta)$) are superanalytic transformations (1). The transformation $\mathcal{G}$ plays the role of the linear transformation of Wess-Zumino type and the nonlinear transformation $\mathcal{H}$ (from a subgroup) is of Akulov-Volkov type, while $\mathcal{A}$ and $\mathcal{B}$ correspond to the transformations with Goldstone fields as parameters (Volkov (1969), Volkov (1973), Volkov and Akulov (1972)).

Global 2D supersymmetry. According to the general prescriptions (Ivanov and Kapustnikov (1978)) we can take $\mathcal{G}$ as a global linear supersymmetry transformation in two-dimensional case

$$\mathcal{G}:\begin{cases}\tilde{z}=z_A+\theta_A\cdot\varepsilon,\\ \tilde{\theta}=\varepsilon+\theta_A,\end{cases} \tag{9}$$

then we take $\mathcal{H}$ as an ordinary conformal transformation with composite parameters to be find and interpret $\mathcal{A}$ and $\mathcal{B}$ as coset transformations with the local odd parameters $\lambda(z)$ and $\tilde{\lambda}_{Glob}(z_H)$

$$\mathcal{A}:\begin{cases}z_A=z+\theta\cdot\lambda(z),\\ \theta_A=\lambda(z)+\theta,\end{cases}\quad \mathcal{B}:\begin{cases}\tilde{z}=z_H+\theta_H\cdot\tilde{\lambda}_{Glob}(z_H),\\ \tilde{\theta}=\tilde{\lambda}_{Glob}(z_H)+\theta_H.\end{cases} \tag{10}$$

Indeed, the commutativity of the diagram gives us the equation of $\lambda(z)$ evolution similar to (3) and (5) and equations for parameters of $\mathcal{H}$ in the following way. A "linear" transformation $\mathcal{G}$ is *representable* by a "nonlinear" transformation $\mathcal{H}$, iff the diagram is commutative

$$\mathcal{G}\circ\mathcal{A}=\mathcal{B}\circ\mathcal{H}. \tag{11}$$

In the group theory this construction is related to the induced representation (Kirillov (1976)). But here we, in general, do not demand invertibility of the entries in (11) and consider finite transformations. Using (11) we obtain the relations

$$\begin{aligned}\tilde{z}_{\mathcal{G}\circ\mathcal{A}}&=\tilde{z}_{\mathcal{B}\circ\mathcal{H}},\\ \tilde{\theta}_{\mathcal{G}\circ\mathcal{A}}&=\tilde{\theta}_{\mathcal{B}\circ\mathcal{H}}\end{aligned} \tag{12}$$

which are the representability condition (11) in coordinate language (as 4 component equations after expanding in θ).

In the particular case of global supersymmetry (9) the equations (12) and (10) give the parameters of the conformal transformation

$$\mathcal{H}:\begin{cases}z_H=z+\lambda(z)\cdot\varepsilon,\\ \theta_H=\theta,\end{cases} \tag{13}$$

and the evolution equation for $\tilde{\lambda}_{Glob}(z_H)=\varepsilon+\lambda(z)$. Then expanding on nilpotents

$$\varepsilon+\lambda(z)=\tilde{\lambda}_{Glob}(z)+\tilde{\lambda}'_{Glob}(z)\cdot\lambda(z)\cdot\varepsilon \tag{14}$$

which coincides with (5).

The $-\lambda$-rule in 2D. If $\mathcal{A}$ is invertible, the representability condition (11) becomes

$$\mathcal{G}=\mathcal{B}\circ\mathcal{H}\circ\mathcal{A}^{-1}. \tag{15}$$

In the global case invertibility of $\mathcal{A}$ is evident, then from (10) we derive

$$\mathcal{A}^{-1}:\begin{cases} z = z_A - \theta_A \cdot \lambda\left(z_A\right), \\ \theta = -\lambda\left(z_A\right) + \theta_A\left[1 + \lambda\left(z_A\right) \cdot \lambda'\left(z_A\right)\right]. \end{cases} \tag{16}$$

This explains nature of the well-known "$-\lambda$ rule" (Ivanov and Kapustnikov (1978)) while comparing superfields of linear and nonlinear realizations (Wess (1983)). The relation (15) is a general form of the "splitting trick" (Ivanov and Kapustnikov (1978), Ivanov and Kapustnikov (1982)) according to which any linear superfield can be presented as a set of nonlinear transforming components. The analog of this trick for a noninvertible finite case is the representability condition (11), and it is not solved under $\mathcal{A}$. Thus, for a superfield $\Phi\left(z,\theta\right)$ we can write $\delta_{\mathcal{H}}\Phi\left(z,\theta\right) = \varepsilon \cdot \lambda\left(z\right) \cdot \frac{\partial \Phi(z,\theta)}{\partial z}$,where $\delta_{\mathcal{H}}$ is infinitesimal "nonlinear" transformation $\mathcal{H}$ corresponding to $\mathcal{G}$. If we use (16) and put $\Phi\left(z,\theta\right) = \Phi_A\left(z_A,\theta_A\right)$,then for infinitesimal "linear" transformation $\mathcal{G}$ we obtain the standard supersymmetry relation

$$\delta_{\mathcal{G}}\Phi_A\left(z_A,\theta_A\right) = \Phi\left(z_A + \varepsilon \cdot \theta_A, \theta_A + \varepsilon\right) - \Phi_A\left(z_A,\theta_A\right) = \varepsilon \cdot Q_A \Phi_A\left(z_A,\theta_A\right), \tag{17}$$

where Q_A is an ordinary supertranslation (Ivanov and Kapustnikov (1978)). Now we are ready to prove the "reversed" splitting trick which manifestly follows from the representability condition (11) applied to global two dimensional supersymmetry. It can be shown that a superfield $\Phi\left(z,\theta\right)$ transforming nonlinearly together with $\lambda\left(z\right)$ transforming as in (6) gives a linearly (globally) transformed superfield (17). Indeed, we see that $\Delta\Phi\left(z,\theta\right) = \delta_{\mathcal{G}}\Phi_A\left(z_A,\theta_A\right)$, where $\Delta\Phi\left(z,\theta\right) \stackrel{def}{=} \delta_{\mathcal{H}}\Phi\left(z,\theta\right) + \delta_{\mathcal{B}}\Phi\left(z,\theta\right) - \delta_{\mathcal{A}}\Phi\left(z,\theta\right)$.

Nonlinear realization of general finite $N = 1$ superconformal transformations. Let us consider the representability condition (11) for a general $N = 1$ superconformal-like transformations $Z_A \to \tilde{Z}$ which now play the role of "linear" ones. According to (Duplij (1996), Duplij (1997)) they can be parametrized by two functions $g\left(z_A\right)$ and $\psi\left(z_A\right)$ and have the form

$$\mathcal{G}:\begin{cases} \tilde{z} = f_n^{(g\psi)}\left(z_A\right) + \theta_A \cdot \chi_n^{(g\psi)}\left(z_A\right), \\ \tilde{\theta} = \psi\left(z_A\right) + \theta_A \cdot g\left(z_A\right), \end{cases} \tag{18}$$

where

$$\begin{aligned} f_n^{(g\psi)\prime}\left(z_A\right) &= \psi'\left(z_A\right)\psi\left(z_A\right) + \tfrac{1+n}{2} \cdot g^2\left(z_A\right), \\ \chi_n^{(g\psi)\prime}\left(z_A\right) &= g'\left(z_A\right)\psi\left(z_A\right) + n \cdot g\left(z_A\right)\psi'\left(z_A\right), \end{aligned} \tag{19}$$

where $n = \begin{cases} +1, \text{ SCf transformation,} \\ -1, \text{ TPt transformation,} \end{cases}$ is a projection of "reduction spin" switching the type of transformation (see (Duplij (1997)) for more details).

Then while trying to represent $\mathcal{G}$ in terms of nonlinear compositions we face with the following restriction which is consequence of the $N = 1$ superconformal-like multiplication law (Duplij (1997)). If $\mathcal{T}$ is a superconformal-like transformation, then there are only two possibilities in the composition $z \xrightarrow{\mathcal{T}} \tilde{z} \xrightarrow{\tilde{\mathcal{T}}} \tilde{\tilde{z}}$

$$\tilde{\mathcal{T}}_{SCf} * \mathcal{T}_{SCf} = \tilde{\tilde{\mathcal{T}}}_{SCf}, \qquad (20)$$
$$\tilde{\mathcal{T}}_{TPt} * \mathcal{T}_{SCf} = \tilde{\tilde{\mathcal{T}}}_{TPt}.$$

Therefore, we have only two possibilities to include TPt transformations into the diagrammatic representation, viz.

$$\mathcal{G}_{SCf} \circ \mathcal{A}_{SCf} = \mathcal{B}_{SCf} \circ \mathcal{H}_{SCf}, \qquad (21)$$

$$\mathcal{G}_{TPt} \circ \mathcal{A}_{SCf} = \mathcal{B}_{TPt} \circ \mathcal{H}_{SCf}. \qquad (22)$$

The first one is the nonlinear representation of $N = 1$ superconformal group in analogy with the ordinary infinitesimal invertible four-dimensional case (Ivanov and Kapustnikov (1978), Ivanov and Kapustnikov (1990)) (and 11) in which $\mathcal{A}_{SCf}$ and $\mathcal{B}_{SCf}$ play the role of cosets.

Let us consider (21) in more detail. The exact shape of cosets $\mathcal{A}_{SCf}$ and $\mathcal{B}_{SCf}$ can be taken as

$$\mathcal{A}_{SCf} : \begin{cases} z_A = z + \theta \cdot \lambda(z), \\ \theta_A = \lambda(z) + \theta\sqrt{1 + \lambda(z) \cdot \lambda'(z)}, \end{cases} \qquad (23)$$

$$\mathcal{B}_{SCf} : \begin{cases} \tilde{z} = z_H + \theta_H \cdot \tilde{\lambda}(z_H), \\ \tilde{\theta} = \tilde{\lambda}(z_H) + \theta_H\sqrt{1 + \tilde{\lambda}(z_H) \cdot \tilde{\lambda}'(z_H)}, \end{cases} \qquad (24)$$

and for $\mathcal{H}$ we choose the following general parametrization

$$\mathcal{H}_{SCf} : \begin{cases} z_H = p(z), \\ \theta_H = \rho(z) + \theta \cdot q(z) \end{cases} \qquad (25)$$

Then, expanding the coordinate form (12) into components we obtain 4 corresponding equations for 4 unknown functions $p(z), q(z), \rho(z), \tilde{\lambda}(z)$

$$p(z) + \rho(z) \cdot \tilde{\lambda}(p(z)) = f_{+1}^{(g\psi)}(z) + g(z) \cdot \lambda(z) \cdot \psi(z), \qquad (26)$$

$$\tilde{\lambda}(p(z)) + \rho(z) \cdot \sqrt{1 + \tilde{\lambda}(p(z)) \cdot \tilde{\lambda}'(p(z))} = \psi(z) + g(z) \cdot \lambda(z), \qquad (27)$$

$$q(z) \cdot \tilde{\lambda}(p(z)) = \lambda(z) \cdot f_{+1}^{(g\psi)\prime}(z) + g(z) \cdot \psi(z) \cdot \sqrt{1 + \lambda(z) \cdot \lambda'(z)}, \qquad (28)$$

$$q(z) \cdot \sqrt{1 + \tilde{\lambda}(p(z)) \cdot \tilde{\lambda}'(p(z))} = \lambda(z) \cdot \psi'(z) + g(z) \cdot \sqrt{1 + \lambda(z) \cdot \lambda'(z)}, \qquad (29)$$

where $f_{+1}^{(g\psi)}(z)$ is determined from (19).

In case $q(z)$ and $g(z)$ are invertible, these equations have the following solution for parameters of nonlinear $\mathcal{H}$ transformation in terms of parameters of "linear" $\mathcal{G}$ transformation as

$$p(z) = f_{+1}^{(g\psi)}(z) + g(z)\cdot\lambda(z)\cdot\psi(z), \tag{30}$$

$$q(z) = \sqrt{p'(z)}, \tag{31}$$

$$\rho(z) = 0, \tag{32}$$

and for goldstino transformation rule

$$\tilde{\lambda}(p(z)) = \psi(z) + g(z)\cdot\lambda(z), \tag{33}$$

that naturally coincides with the previous approach (4) with $f(z) = f_{+1}^{(g\psi)}(z)$ and $\chi(z) = g(z)\cdot\psi(z)$.

Therefore, $\mathcal{H}$ is the split $N = 1$ SCf transformation (Friedan (1986))

$$\mathcal{H}_{SCf} : \begin{cases} z_H = p(z), \\ \theta_H = \theta\cdot\sqrt{p'(z)} \end{cases} \tag{34}$$

with the composite parameter $p(z)$ from (30), which can be presented as the following commutative diagram

$$\begin{array}{ccc} Z_A & \xrightarrow[\text{full}]{\mathcal{G}_{SCf}} & \tilde{Z} \\ \mathcal{A}_{SCf}\Big\uparrow & & \Big\uparrow\mathcal{B}_{SCf} \\ Z & \xrightarrow[\text{split}]{\mathcal{H}_{SCf}} & Z_H \end{array}$$

Second relation (22) and the corresponding commutative diagram

$$\begin{array}{ccc} Z_A & \xrightarrow{\mathcal{G}_{TPt}} & \tilde{Z} \\ \mathcal{A}_{SCf}\Big\uparrow & & \Big\uparrow\mathcal{B}_{TPt} \\ Z & \xrightarrow{\mathcal{H}_{SCf}} & Z_H \end{array}$$

have no such transparent meaning, because $\mathcal{B}_{TPt}$ is noninvertible, and so it cannot be a standard coset. Nevertheless, since the final answer for the nonlinear transformation $\mathcal{H}_{SCf}$ is known from the previous approach (7), the noninvertible analog of coset $\mathcal{B}_{TPt}$ can be found in principle from the system of equations analogous to (26)–(29).

Let we write $\mathcal{B}_{TPt}$ in the form

$$\mathcal{B}_{SCf} : \begin{cases} \tilde{z} = f_{-1}^{(\widetilde{b\lambda})}(z_H) + \theta_H\cdot\chi_{-1}^{(\widetilde{b\lambda})}(z_H), \\ \tilde{\theta} = \tilde{\lambda}(z_H) + \theta_H\cdot b(z_H), \end{cases} \tag{35}$$

where

$$\begin{aligned} f_n^{(b\widetilde{\lambda})\prime}(z_H) &= \widetilde{\lambda}'(z_H)\cdot\widetilde{\lambda}(z_H)+\tfrac{1+n}{2}\cdot b^2(z_H), \\ \chi_n^{(b\widetilde{\lambda})\prime}(z_H) &= b'(z_H)\cdot\widetilde{\lambda}(z_H)+n\cdot b(z_H)\cdot\widetilde{\lambda}'(z_H), \end{aligned} \tag{36}$$

and prime denotes derivative by argument. So the corresponding system of equations now is

$$f_{-1}^{(b\widetilde{\lambda})}(p(z))+\rho(z)\cdot\chi_{-1}^{(b\widetilde{\lambda})}(p(z)) = f_{+1}^{(g\psi)}(z)+\lambda(z)\cdot\chi_{+1}^{(g\psi)}(z), \tag{37}$$

$$\widetilde{\lambda}(p(z))+\rho(z)\cdot b(p(z)) = \psi(z)+g(z)\cdot\lambda(z), \tag{38}$$

$$\begin{aligned}\rho(z)\cdot f_{-1}^{(b\widetilde{\lambda})\prime}(p(z))+q(z)\cdot\chi_{-1}^{(b\widetilde{\lambda})}(p(z)) = \lambda(z)\cdot f_{+1}^{(g\psi)\prime}(z)+ \\ \chi_{+1}^{(g\psi)}(z)\cdot\sqrt{1+\lambda(z)\cdot\lambda'(z)},\end{aligned} \tag{39}$$

$$\begin{aligned}\rho(z)\cdot q(z)\cdot\widetilde{\lambda}'(p(z))+q(z)\cdot b(p(z)) = \lambda(z)\cdot\psi'(z)+ \\ g(z)\cdot\sqrt{1+\lambda(z)\cdot\lambda'(z)}.\end{aligned} \tag{40}$$

In case $\mathcal{A}_{SCf}$ is invertible we can obtain $\mathcal{G}_{TPt}=\mathcal{B}_{TPt}\circ\mathcal{H}_{SCf}\circ\mathcal{A}_{SCf}^{-1}$ which gives an analog of nonlinear realization for noninvertible TPt transformations.

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